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Your Roll No.....

Sr. No. of Question Paper : 897 I
Unique Paper Code : 32355101
Name of the Paper : Calculus
Name of the Course : **Mathematics : G.E. for Honours**
Semester : I

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Do any **five** questions from each of the **three** sections.
3. Each question is for **five** marks.

SECTION 1

1. Given $f(x) = 2x - 2$, $x_0 = -2$, $\varepsilon = 0.02$. Find $L = \lim_{x \rightarrow x_0} f(x)$.
Then find a number $\delta > 0$ such that for all x , $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.
2. Find a linearization of $f(x) = \sqrt{x^2 + 9}$ at $x = -4$.

P.T.O.

3. The radius of a circle is increased from 2.00 to 2.02 m. Estimate the resulting change in area. Also express the estimate as a percentage of the circle's original area.

4. Evaluate the limit $\lim_{x \rightarrow 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$.

5. Use L'Hôpital's rule to find $\lim_{x \rightarrow 0^+} \frac{\ln(e^x - 1)}{\ln x}$.

6. Sketch the graph of a function $f(x) = x^3 - 3x + 1$.

7. Find the volume of the solid generated by revolving the region between the y-axis and curve $x = 2/y$, $1 \leq y \leq 4$, about the y-axis.

SECTION 2

8. Use the shell method to find the volume of the solid generated when the region R in the first quadrant enclosed between $y = x$ and $y = x^2$ is revolved about the y-axis.

9. Sketch the graph of $r = 1 - 2\cos\theta$ and identify its symmetry.

10. Find the area of the surface generated by revolving the curve $y = \sqrt{x}$, $0 \leq x \leq 1$, about the x-axis.

Suppose a person on a hang glider is spiraling upward due to rapidly rising air on a path having acceleration vector

$a(t) = -3\cos t \mathbf{i} - 3\sin t \mathbf{j} + 2\mathbf{k}$. It is also known that initially (at time $t=0$), the glider departed from the point $(3,0,0)$ with velocity $v(0) = 3\mathbf{j}$. Find the glider's position as a function of t .

Find the unit tangent vector of the curve

$$r(t) = t^2 \mathbf{i} + 2\cos t \mathbf{j} + 2\sin t \mathbf{k}.$$

Determine whether $\int_{-\infty}^{-1} \frac{1}{x} dx$ converges?

Find the arc length parameterization of the helix

$$r(t) = \cos 4t \mathbf{i} + \sin 4t \mathbf{j} + 3t \mathbf{k} \quad 0 \leq t \leq 2\pi.$$

SECTION 3

Show that the ellipse $x = a \cos t$, $y = b \sin t$, $a > b > 0$, has its largest curvature on its major axis and its smallest curvature on its minor axis.

Find the binormal vector \vec{B} and the torsion function τ for the space curve

$$\vec{r}(t) = (\cos t + t \sin t) \hat{i} + (\sin t - t \cos t) \hat{j} + 3t \hat{k}$$

P.T.O.

17. Show that the function

$$f(x, y) = \frac{xy}{|xy|}$$

has no limit as (x, y) approaches $(0, 0)$.

18. If $w = \ln(x^2 + y^2 + z^2)$, $x = ue^v \sin u$, $y = ue^v \cos u$

$z = ue^v$, find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ using chain rule at the point

$$= (-2, 0).$$

19. Find the directions in which the function $f(x, y, z) = \ln xy + \ln yz + \ln xz$ increase and decrease most rapidly at the point $P_0(1, 1, 1)$. Then find the derivative of the function in those directions.

20. Find parametric equations for the line tangent to the curve of intersection of the surfaces

$$x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0 \quad \text{and} \quad x^3 + y^2 + z^2 = 11$$

at the point $(1, 1, 3)$.

21. Find the absolute maxima and minima of the function

$$T(x, y) = x^2 + xy + y^2 - 6x + 2$$

on the rectangular plate $0 \leq x \leq 5$; $-3 \leq y \leq 0$.